**Suggested solution of HW1**

1. (a)

$$\begin{aligned} a \cdot 0 &= a \cdot (0+0) \\ &= a \cdot 0 + a \cdot 0. \end{aligned}$$

   Hence $a \cdot 0 = 0$.

   (b)

$$a + (-1)a = a + a \cdot (-1) = a \cdot [1 + (-1)] = a \cdot 0 = 0.$$

   By uniqueness, $(-1)a = -a$.

2. If $x - a \geq 0$, then

$$x - a = |x - a| < \epsilon \Longrightarrow a \leq x < a + \epsilon.$$

   If $x - a < 0$, then

$$a - x = |x - a| < \epsilon \Longrightarrow a - \epsilon \leq x < a.$$

   Result follows from combining two cases.

3. Let $A$ be a nonempty subset of $\mathbb{R}$ and $l \in \mathbb{R}$. $l$ is said to be a lower bound of $A$, if

$$l \leq a, \quad \forall\, a \in A.$$

   Negation: $l$ is not a lower bound of $A$ if

$$a < l, \quad \exists\, a \in A.$$

4. $x_n \to x, y_n \to y$ if and only if for any $\epsilon > 0$, there exists $N \in \mathbb{N}$ such that for all $n \geq N$,

$$|x_n - x| < \epsilon/2 \quad \text{and} \quad |y_n - y| < \epsilon/2.$$

   Then

$$|(x_n + y_n) - (x + y)| \leq |x_n - x| + |y_n - y| < \epsilon.$$